9. Spins and parities of states in odd-odd nuclei (Krane 5.5)
In the single-particle shell model, the ground state and often several low-lying excited states of a nucleus with an odd proton and an odd neutron are determined from the coupling of the proton and neutron shell-model states $\vec{I} = \vec{j}_p + \vec{j}_n$.

(a) Consider the following nuclei: $^{16}\text{N}$ ($I^\pi = 2^-$), $^{26}\text{Al}$ ($I^\pi = 5^+$), and $^{26}\text{Na}$ ($I^\pi = 3^+$). Draw simple vector diagrams illustrating these couplings, then replace $\vec{j}_p$ and $\vec{j}_n$, respectively, by $\vec{l}_p + \vec{s}_p$ and $\vec{l}_n + \vec{s}_n$.

(b) Examine your diagrams and deduce an empirical rule for the relative orientation of $\vec{s}_p$ and $\vec{s}_n$ in the ground state.

(c) Use your empirical rule to predict the $I^\pi$ assignments of $^{28}\text{Al}$ and $^{30}\text{P}$.

10. Two-state mixing
In realistic calculations of nuclear states pure configurations are rare. The states are complex admixtures of many components. A feeling for the underlying physics can be obtained from a simple two-state mixing calculation. Consider the situation illustrated in the Figure. Two initial (pure) states have the energies $E_1$ and $E_2$ and the wave functions $\Phi_1$ and $\Phi_2$. For an arbitrary interaction, $V$, the mixing matrix element is $\langle \Phi_1 | V | \Phi_2 \rangle$, which we denote simply as $V_{12}$. The final energies $E_I$ and $E_{II}$ and wave functions $\Psi_I$ and $\Psi_{II}$ are obtained by diagonalizing the $2 \times 2$ matrix

$$
\begin{pmatrix}
E_1 & V_{12} \\
V_{12} & E_2
\end{pmatrix}
$$

The final quantities are denoted by Roman numerals.

(a) With $\Delta E_{12} = E_1 - E_2$ (see Figure) and $R = \Delta E_{12}/V_{12}$ show that

$$
\Delta E_{I,II} = E_{II} - E_I = \Delta E_{12} \sqrt{1 + \frac{4}{R^2}}
$$

(b) Calculate $\Delta E_{I,II}$ for the four combinations of $\Delta E_{12} = 70$ keV and 190 keV and $V_{12} = 40$ keV and 130 keV, respectively. Discuss your results.
11. Isotope production reaction (Krane 11.14)
The radioactive isotope $^{15}\text{O}$, which has important medical applications, can be produced in the nuclear reaction $^{12}\text{C}(\alpha, n)$.
(a) The cross section reaches a peak when the laboratory energy of the incident $\alpha$ particle is 14.6 MeV. What is the excitation energy of the compound nuclear state?
(b) The reaction cross section at the above incident energy is 25 mb. Assuming a carbon target of 0.10 mg/cm$^2$ and a current of 20 nA of the $^4\text{He}^{2+}$ ions (= $\alpha$ particles), compute the $^{15}\text{O}$ activity that results after 4 minutes of irradiation.
Hint: check also Krane Chapter 6.3.

12. Thermal neutron flux (Krane 12.11)
The intensity of a source of thermal neutrons is to be measured by counting the induced radioactivity in a thin foil of indium ($Z = 49$) metal exposed to the neutrons. The foil is 3.0 $\times$ 3.0 mm$^2$ in area and 1.0 $\mu$m in thickness. The activation of $^{115}\text{In}$ to $^{116}\text{In}$ takes place with a cross section of 160 b for thermal neutrons. The half-life of $^{116}\text{In}$ is $T_{1/2} = 54$ min. The foil is irradiated for one minute, but after the conclusion of the irradiation, the counting of the decays from the foil cannot be started for 30 minutes. The efficiency of the detection system is only $\epsilon = 2.4 \cdot 10^{-4}$. In one hour of counting, $4.85 \cdot 10^4$ counts are accumulated.
What is the thermal neutron flux?

13. Coulomb scattering
(a) In Coulomb scattering of 7.50 MeV protons by a target of $^7\text{Li}$, what is the energy of the elastically scattered protons at $\theta = 45^\circ$, $90^\circ$, and $135^\circ$?
(b) What is the energy of the inelastically scattered protons at $\theta = 90^\circ$, when the nucleus $^7\text{Li}$ is left in its first excited state at $E_x = 0.477$ MeV?
(c) Calculate the incident energy of a proton beam to be Coulomb scattered by gold nuclei, when it is desired that the minimum distance between projectile and target should correspond to the two nuclei just touching at their surface?
(d) Do the same calculation as in (c), but for the system $^{48}\text{Ca}$ beam on $^{243}\text{Am}$ target nuclei.
(e) Bonus question: What is the excitation energy of the $^{291}\text{Mc}$ compound nucleus produced with the reaction mentioned in (d) at a beam energy of $E^{(48}\text{Ca}) = 245.0$ MeV?

14. Time-of-flight measurement of fast neutrons
The schematic below shows the (simplified!) setup for a time-of-flight (TOF) measurement from a source emitting both fast neutrons and $\gamma$ rays. Both types of radiation travel a distance $d = 1.1$ m before being seen by a detector.

The goal of the experiment is to determine the energy of a neutron by measuring the time between its emission and its detection, $t_{\text{TOF}}$. We assume that we have a well-defined, fixed
reference point, \( t_{\text{start}} \), at which we always start our stop watch with respect to the actual emission of either neutron or \( \gamma \) at \( t_0 \). (This reference can be, for example, the detection of an additional process that coincides with the emission of the neutron or \( \gamma \) ray – for more details, see the neutron tagging laboratory!).

For this exercise, the detector is only used to provide the stopping time, \( t_{\text{stop}} \).

The plot below shows the number of counts as function of the stopping time as measured by the detector. Besides the background of around 10 counts per bin, you notice two distinct peak structures: one narrow peak caused by \( \gamma \) rays hitting the detector and one broad ‘shark fin’ distribution due to incident neutrons.

(a) Calibration of the setup
Since our \( t_{\text{start}} \) is somewhat arbitrarily set and we have an unknown amount of cables and electronics in our setup which delay the signals, the actual \( t_0 \) of emission is not at \( x = 0 \). We have to first determine this shift. However, since we know the distance \( d \), and \( \gamma \) rays always travel at the speed of light, we can calculate \( t_0 \) from the position of the peak associated with the \( \gamma \) ray detection. Read the position of the ‘\( \gamma \) peak’ off the plot and calculate when the \( \gamma \) rays have been emitted (\( \equiv t_0 \)).

(b) Calculating the time-of-flight
With \( t_0 \) as reference, read off the plot the TOF for both the fastest and the slowest neutrons.

(c) Calculating the kinetic energy of the neutrons
Use the TOF determined above to calculate the kinetic energy, \( T_n \), of both the fastest and the slowest of the measured neutrons in units of MeV. Hint: in the energy range this data was taken, \( T_n \ll m_n \), where \( m_n \) is the rest mass of the neutron.